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Identification of wall heat flux for turbulent forced convection by inverse analysis

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Abstract

An inverse problem for turbulent forced convection between parallel flat plates is investigated. The space- and time-dependent heat flux at the upper wall is estimated from the temperature measurements taken inside the flow. In the present study, the conjugate gradient method is adopted for the estimation of the unknown wall heat flux. No prior information is needed for the functional form of the wall heat flux in the inverse analysis. The effects of the measurement errors, the functional form of the wall heat flux, and the location of the sensors on the accuracy of the estimation are investigated. The reconstruction of the wall heat flux is satisfactory when simulated exact or noisy data are input to the inverse analysis. The sensitivity coefficients are discussed in this paper. As expected, it is shown that the accuracy of the estimation can be improved when the sensors are located closer to the upper wall.

1. Introduction

Inverse heat transfer problems are important when direct measurements of the desired physical quantities are not possible. The inverse techniques can be used to determine crucial parameters in conduction, convection, and radiation in many engineering applications. Examples include, among others, estimation of the boundary heat flux of a heating material, control of the freezing interface of a solidifying metal, and determination of the radiative properties of a semi-transparent medium. The inverse problems are known as ill-posed, hence the estimation is very sensitive to the measurement errors of the input data. To overcome the instability of the inverse problem different methods have been developed. Several texts have been devoted to this topic [1–4].

Inverse problems of heat convection started to receive much attention more recently. Raghunath [5], Bokar and Ozisik [6], and Liu and Ozisik [7] considered the inverse convection problem of determining the inlet temperature of a thermally developing hydrodynamically developed laminar flow between parallel plates from temperature measurements taken downstream of the entrance. Moutsoglou [8] investigated the steady-state inverse forced convection problem between parallel flat plates. The wall heat flux of the top wall was estimated from measured temperature data at the bottom wall using the straight inversion and the whole domain regularization schemes. Huang and Ozisik [9] determined the spacewise variation of the wall heat flux for laminar flow in a parallel plate duct from temperature measurements inside the flow at several different locations along the flow. Liu and Ozisik [10] estimated the timewise variation of the wall heat flux for transient turbulent forced convection inside parallel plate ducts. The conjugate gradient method with an adjoint equation was adopted to solve the problem. Machado and Orlande [11] applied the conjugate gradient method with an adjoint equation to estimate the timewise and spacewise variation of the wall heat flux in laminar forced convection. Park and Lee [12] employed the Karhunen-Loeve Galerkin procedure to solve the inverse problem of determining the space-dependent wall heat flux for laminar flow inside a duct from the temperature measurement within the flow.

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Nomenclature

b	channel width
d	direction of descent
J	objective function
k	thermal conductivity
M	the number of the measured data in the X-direction
N	the number of the measured data in the τ-direction
Pr	Prandtl number
Q	dimensionless wall heat flux
q	wall heat flux
q_{ref}	reference heat flux
Re	Reynolds number
T	temperature
T_0	initial temperature
t	time
U	dimensionless velocity
u_0	inlet velocity
u	velocity
X, Y	dimensionless coordinates
x, y	coordinates
Y_1	Y coordinate of the sensors
Z	measured temperature data

Greek symbols

α	thermal diffusivity
β	step size
ε^+	dimensionless turbulent viscosity
γ	conjugate coefficient
ν	kinematic viscosity
θ	dimensionless temperature
σ	standard deviation
τ	dimensionless time
ζ	random variable

Subscript

t	turbulent

Superscript

p	pth iteration

In the present paper, an inverse problem for the estimation of the space- and time-dependent wall heat flux for unsteady turbulent forced convection between parallel flat plates from the temperature measurements taken inside the flow is considered. The governing equations for the direct problem will be introduced first. The inverse analysis will then be considered. Test cases will be presented to discuss the effects of the measurement errors, the functional form, and the location of the sensors on the estimation.

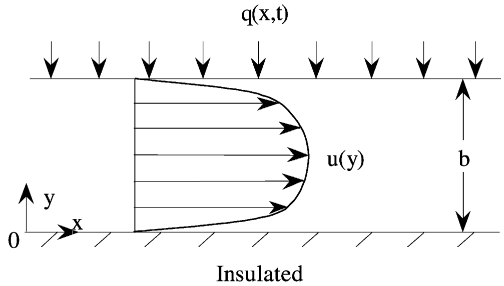

Fig. 1. Schematic of the physical system and coordinates.

2. Analysis

2.1. Direct problem

We consider a thermally developing, hydrodynamically developed turbulent flow through a horizontal plane channel with width b. The flow is assumed to be two-dimensional, and the fluid is Newtonian and constant properties. Initially, the flow and the channel are at the same temperature T_0. Fluid enters the channel at a uniform temperature T_0. At $t > 0$, the lower wall is maintained insulated while the upper wall is subjected to a wall heat flux $q(x,t)$. A schematic diagram of the problem is given in Fig. 1. The axial conduction and viscous dissipation are ignored. The governing equation in dimensionless form is given by

$$\frac{\partial\theta}{\partial\tau}+\frac{1}{2}PrReU\frac{\partial\theta}{\partial X}=\frac{\partial}{\partial Y}\left[\left(1+\frac{Pr}{Pr_{\mathrm{t}}}\varepsilon^{+}\right)\frac{\partial\theta}{\partial Y}\right] \tag{1}$$

with the initial and boundary conditions

$$\theta(X,Y,0)=0 \tag{2}$$

$$\theta(0,Y,\tau)=0 \tag{3}$$

$$\frac{\partial\theta(X,0,\tau)}{\partial Y}=0 \tag{4}$$

$$\frac{\partial\theta}{\partial Y}(X,1,\tau)=Q(X,\tau) \tag{5}$$

where

$$\begin{aligned}&X=\frac{x}{b},\quad Y=\frac{y}{b},\quad U=\frac{u}{u_0},\quad \tau=\frac{\alpha t}{b^2}\\&Re=\frac{2U_0b}{\nu},\quad \theta=\frac{k(T-T_0)}{q_{\mathrm{ref}}b},\quad Q=\frac{q}{q_{\mathrm{ref}}}\\&Pr=\frac{\nu}{\alpha},\quad Pr_{\mathrm{t}}=\frac{\nu_{\mathrm{t}}}{\alpha_{\mathrm{t}}},\quad \varepsilon^{+}=\frac{\nu_{\mathrm{t}}}{\nu}\end{aligned} \tag{6}$$

Here, ν is the kinematic viscosity, α is the thermal diffusivity, k is the thermal conductivity, u_0 is the inlet velocity, ν_t is the turbulent diffusivity of momentum, α_t is the turbulent diffusivity of heat, q_{ref} is the reference heat flux. Heat transfer into the fluid is assumed to be positive. The fully developed turbulent velocity profile u and turbulent diffusivity of momentum ν_t are solved by the low Reynolds number k-ε model used in Ref. [13]. The value of the turbulent Prandtl number Pr_t is taken to be 0.9.

The direct problem can be solved to obtain the dimensionless temperature field when U, Q, Re, Pr, Pr_t and ε^+ are known. A fully implicit numerical scheme in which the x-direction convection term is approximated by the upstream difference, the y-direction diffusion term by the central difference and the unsteady term by the backward difference is employed to transform the governing equations into finite difference equations. This system of equations forms a tridiagonal matrix which can be solved by the Thomas Algorithm [14].

2.2. Inverse problem

The inverse problem considered in this paper is to estimate the unknown dimensionless wall heat flux from temperature measurements taken in the flow field. It is solved as an optimization problem which minimizes the summation of the square of the differences between the estimated dimensionless temperatures $\theta(X_i, Y_1, \tau_k)$ and the measured dimensionless temperatures $Z(X_i, Y_1, \tau_k)$. The objective function J is given by

$$J = \sum_{i=1}^{M}\sum_{k=1}^{N}(\theta_{i,k} - Z_{i,k})^2 \tag{7}$$

where $\theta_{i,k} = \theta(X_i, Y_1, \tau_k)$, $Z_{i,k} = Z(X_i, Y_1, \tau_k)$, M is the number of the sensors, N is the number of the data sampled for each sensor, and Y_1 is the Y coordinate of the sensors. The minimization of the objective function is obtained by the conjugate gradient method [15]. Iterations are built in the following manner

$$Q_{m,n}^{p+1} = Q_{m,n}^{p} - \beta^p d_{m,n}^{p} \tag{8}$$

where $Q_{m,n} = Q(X_m, \tau_n)$, β^p is the step size, $d_{m,n}^p$ is the direction of descent which is determined from

$$d_{m,n}^{p} = \left(\frac{\partial J}{\partial Q_{m,n}}\right)^p + \gamma^p d_{m,n}^{p-1} \tag{9}$$

and the conjugate coefficient γ^p is computed from

$$\gamma^p = \frac{\sum_{m=1}^{M}\sum_{n=1}^{N}\left[\left(\frac{\partial J}{\partial Q_{m,n}}\right)^p\right]^2}{\sum_{m=1}^{M}\sum_{n=1}^{N}\left[\left(\frac{\partial J}{\partial Q_{m,n}}\right)^{p-1}\right]^2} \quad \text{with } \gamma^0 = 0 \tag{10}$$

The step size is determined from

$$\beta^p = \frac{\sum_{i=1}^{M}\sum_{k=1}^{N}(\theta_{i,k}^p - Z_{i,k})\sum_{m=1}^{M}\sum_{n=1}^{N}\left(\frac{\partial\theta_{i,k}}{\partial Q_{m,n}}\right)^p d_{m,n}^p}{\sum_{i=1}^{M}\sum_{k=1}^{N}\left[\sum_{m=1}^{M}\sum_{n=1}^{N}\left(\frac{\partial\theta_{i,k}}{\partial Q_{m,n}}\right)^p d_{m,n}^p\right]^2} \tag{11}$$

The iteration process given by Eqs. (8)–(11) requires the sensitivity coefficient $\partial\theta_{i,k}/\partial Q_{m,n}$ and the gradient of the objective function $\partial J/\partial Q_{m,n}$.

2.3. Sensitivity problem

The sensitivity problem is obtained by differentiating the direct problem given by Eqs. (1)–(5) with respect to $Q_{m,n}$, from which we can show that

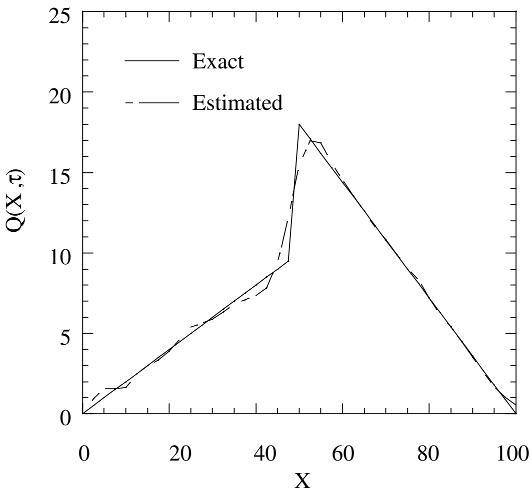

Fig. 2. Exact and estimated wall heat fluxes for $Y_1 = 0.9$ and $\sigma = 0.02$.

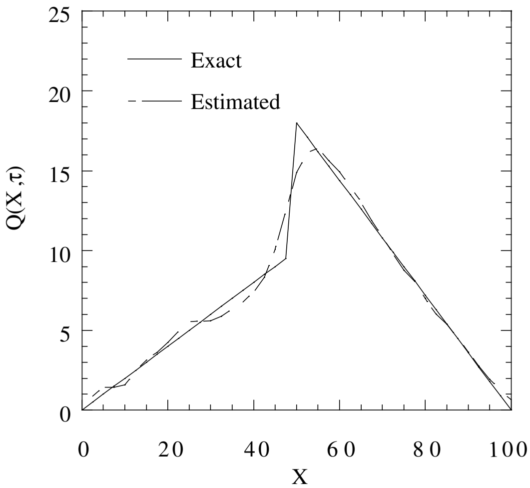

Fig. 3. Exact and estimated wall heat fluxes for $Y_1 = 0.9$ and $\sigma = 0.04$.

$$\frac{\partial}{\partial\tau}\left(\frac{\partial\theta}{\partial Q_{m,n}}\right)+\frac{1}{2}PrReU\frac{\partial}{\partial X}\left(\frac{\partial\theta}{\partial Q_{m,n}}\right)=\frac{\partial}{\partial Y}\left[\left(1+\frac{Pr}{Pr_{\mathrm{t}}}\varepsilon^{+}\right)\frac{\partial}{\partial Y}\left(\frac{\partial\theta}{\partial Q_{m,n}}\right)\right] \tag{12}$$

$$\frac{\partial\theta(X,Y,0)}{\partial Q_{m,n}}=0 \tag{13}$$

$$\frac{\partial\theta(0,Y,\tau)}{\partial Q_{m,n}}=0 \tag{14}$$

$$\frac{\partial}{\partial Y}\left(\frac{\partial\theta(X,0,\tau)}{\partial Q_{m,n}}\right)=0 \tag{15}$$

$$\frac{\partial}{\partial Y}\left(\frac{\partial\theta(X,1,\tau)}{\partial Q_{m,n}}\right)=\widehat{u}(X-X_m,\tau-\tau_n) \tag{16}$$

for $m=1,2,\ldots,M$ and $n=1,2,\ldots,N$, where

$$\widehat{u}(X-X_m,\tau-\tau_n)=\begin{cases}1, & X=X_m,\tau=\tau_n\\ 0, & \text{otherwise}\end{cases} \tag{17}$$

2.4. Gradient equation

The gradient of the objective function, $\partial J/\partial Q_{m,n}$, is determined by differentiating equation [7] with respect to $Q_{m,n}$ to obtain

$$\frac{\partial J}{\partial Q_{m,n}}=2\sum_{i=1}^{M}\sum_{k=1}^{N}(\theta_{i,k}-Z_{i,k})\frac{\partial\theta_{i,k}}{\partial Q_{m,n}} \tag{18}$$

2.5. Stopping criterion

If the problem contains no measurement errors, the condition $J(Q^{p}_{m,n})<\delta$ can be used for terminated the

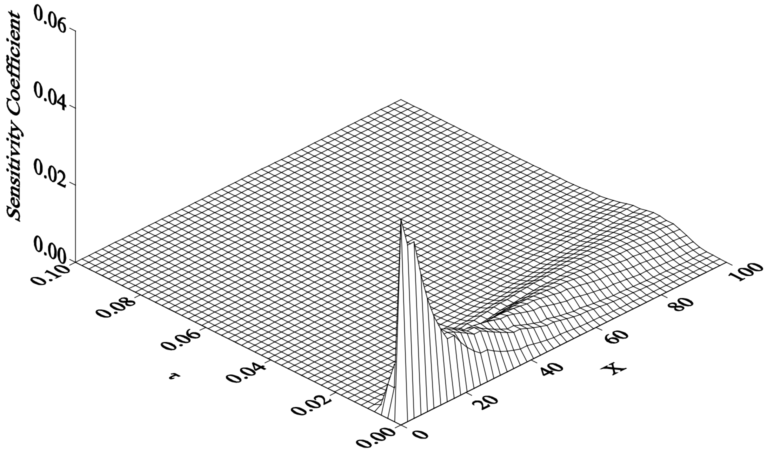

Fig. 4. The sensitivity coefficient $\partial\theta(X,0.95,\tau)/\partial Q_{2,2}$.

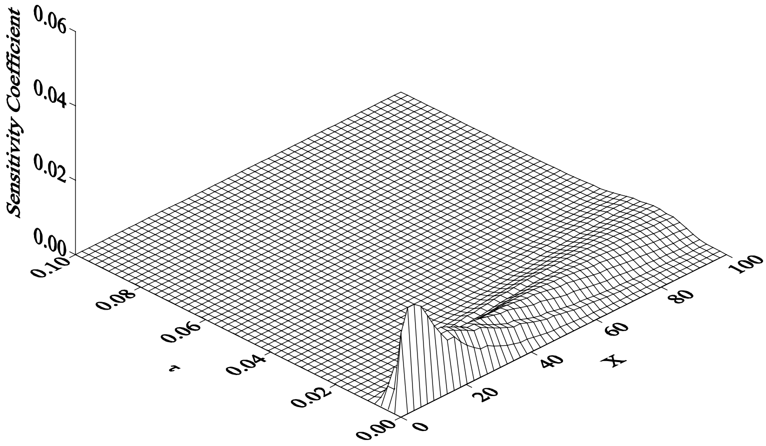

Fig. 5. The sensitivity coefficient $\partial\theta(X,0.9,\tau)/\partial Q_{2,2}$.

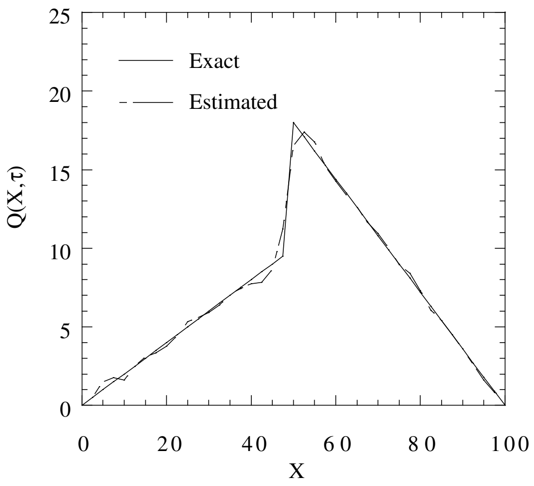

Fig. 6. Exact and estimated wall heat fluxes for $Y_1 = 0.95$ and $\sigma = 0.02$.

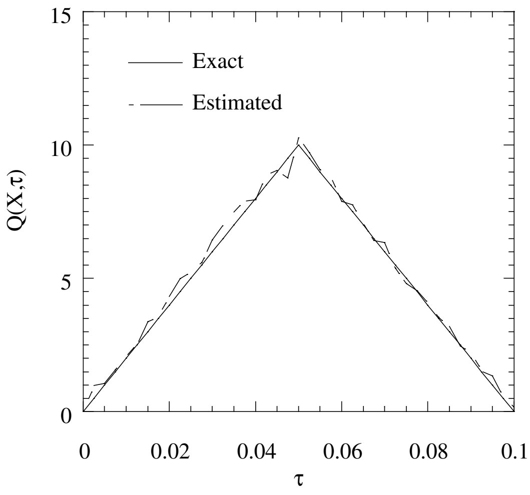

Fig. 8. Exact and estimated wall heat fluxes for $Y_1 = 0.9$ and $\sigma = 0.04$.

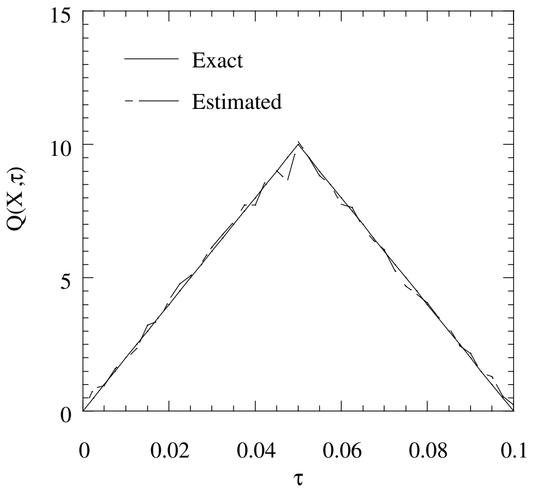

Fig. 7. Exact and estimated wall heat fluxes for $Y_1 = 0.9$ and $\sigma = 0.02$.

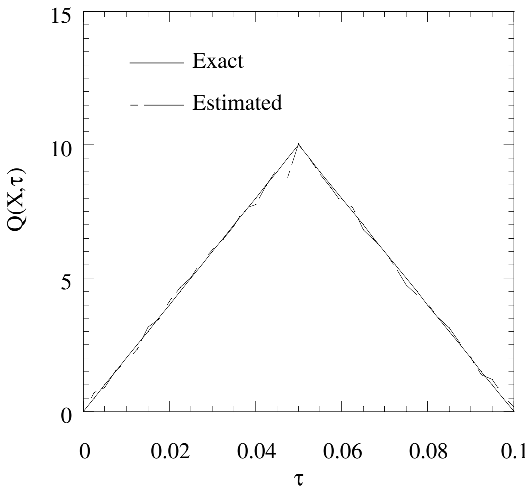

Fig. 9. Exact and estimated wall heat fluxes for $Y_1 = 0.95$ and $\sigma = 0.02$.

iterative process, where δ is a small specified positive number. However, the measured temperature data contain measurement errors. Following the computational experience, we use the discrepancy principle [16] $J(Q^p_{m,n}) < MN\sigma^2$ as the stopped criterion, where σ is the standard deviation of the measurement errors.

2.6. Computational algorithm

The present algorithm for the inverse convection problem is summarized below.

Step 1: Solve the sensitivity problem to calculate the sensitivity coefficient $\partial\theta_{i,k}/\partial Q_{m,n}$.

Step 2: Pick an initial guess $Q^0_{m,n}$ and set $p = 0$.

Step 3: Solve the direct problem to compute $\theta_{i,k}$.

Step 4: Calculate the objective function. Terminate the iteration process if the specified stopping criterion is satisfied. Otherwise go to Step 5.

Step 5: Knowing $\partial\theta_{i,k}/\partial Q_{m,n}$, $\theta_{i,k}$ and $Z_{i,k}$, compute the gradient of the objective function $\partial J/\partial Q_{m,n}$.

Step 6: Knowing $\partial J/\partial Q_{m,n}$, compute γ^p and $d^p_{m,n}$.

Step 7: Knowing $\partial\theta_{i,k}/\partial Q_{m,n}$, $\theta_{i,k}$, $Z_{i,k}$, and $d^p_{m,n}$, compute β^p.

Step 8: Knowing β^p and $d^p_{m,n}$, compute $Q^{p+1}_{m,n}$. Set $p = p + 1$ and go to Step 3.

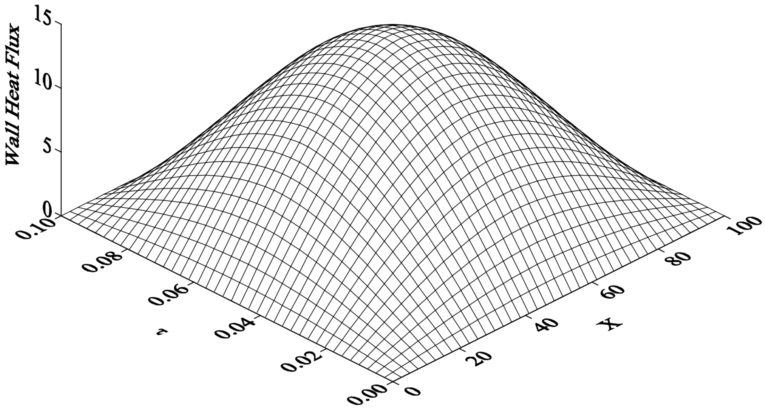

Fig. 10. The exact function for the wall heat flux.

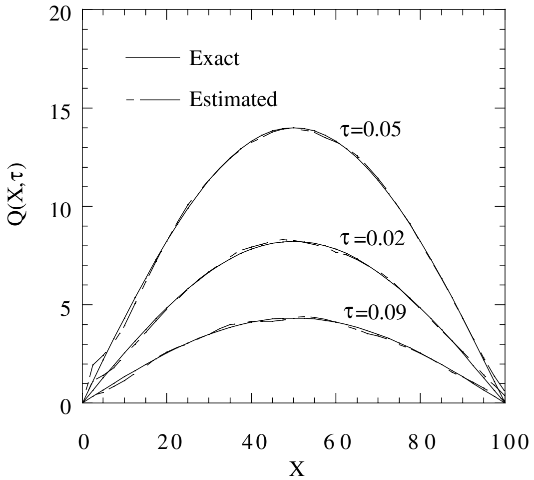

Fig. 11. Exact and estimated wall heat fluxes for $Y_1 = 0.9$ and $\sigma = 0.02$.

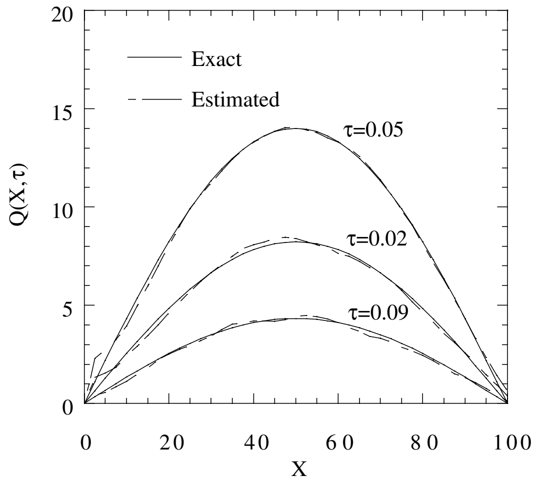

Fig. 12. Exact and estimated wall heat fluxes for $Y_1 = 0.9$ and $\sigma = 0.04$.

3. Results and discussion

To examine the accuracy of the method presented in this paper, three test cases are considered for the estimation of the upper wall heat flux from the simulated measured temperature data. The effects of the measurement errors, the functional form of the wall heat flux, and the location of the sensors on the accuracy of the estimation are investigated. The measurement errors are assumed to be additive, uncorrelated and normally distributed, with known standard deviation and zero mean. The measured temperature data, Z, are simulated by adding random errors to the exact temperature, θ, computed from the solution of the direct problem

$$Z = \theta + \sigma\zeta \tag{19}$$

where σ is the standard deviation of the measurement data, ζ is a random variable of normal distribution with zero mean and unit standard deviation. In the present study, Pr, Pr_t and Re are taken to be 0.7, 0.9 and 20,000, respectively. Forty-one equally spaced measurements are taken both in $0 \leqslant X \leqslant 100$ and $0 \leqslant \tau \leqslant 0.1$ for all the cases considered in this work. The data are used as input to reconstruct the unknown wall heat flux in the inverse problem. The measurement errors become more correlated as the sampling rate increases and the distance between the sensors decreases [1]. High correlation between measurement data provides less information for the inverse analysis. Furthermore, the computational time and experimental expenditures also increase. It is shown that increasing the measurement points from 21×21 to 41×41 improves the accuracy significantly. As a result, 41×41 measurement points are used in this paper.

In the first case, the unknown wall heat flux is assumed to be a function of X only

$$Q(X,\tau) = \begin{cases} 0.2X & 0 \leqslant X < 50 \\ 36 - \frac{9}{25}X & 50 \leqslant X \leqslant 100 \end{cases} \tag{20}$$

The solutions of the inverse analysis from temperature measurements taken inside the flow $Y_1 = 0.9$ for noisy input data $\sigma = 0.02$ is shown in Fig. 2. The deviation between the exact and estimated wall heat fluxes is maximum near the discontinuity, i.e. $X = 50$. The effects of measurement errors are shown in Fig. 3. A comparison of Figs. 2 and 3 shows that the accuracy of the estimation decreases as σ is increased. The sensitivity coefficient is the first derivative of the dimensionless temperature with respective to the unknown wall heat flux. A large sensitivity coefficient indicates that the dimensionless temperature is sensitive to changes in the unknown wall heat flux, while a small sensitivity coefficient implies that the dimensionless temperature is insensitive to changes in the unknown wall heat flux. Figs. 4 and 5 show the sensitivity coefficient $\partial\theta(X,Y,\tau)/\partial Q_{2,2}$ at different sensor locations, i.e. $Y = 0.95$ and 0.9, respectively. The magnitude of the sensitivity coefficient increases as the distance between the upper wall and the sensors decreases. Fig. 6 is used to show the effects of the location of the sensors on the inverse solution. As a result, more accurate solution is obtained when the sensors are closer to the unknown wall heat flux.

In the second case, the wall heat flux is taken to be a function of τ only

$$Q(X,\tau) = \begin{cases} 200\tau & 0 \leqslant \tau \leqslant 0.05 \\ 200(0.1-\tau) & 0.05 \leqslant \tau \leqslant 0.1 \end{cases} \tag{21}$$

Figs. 7–9 show the inverse solutions from the measured temperature data taken inside the flow. The wall heat flux is well reconstructed for all the results. Comparing Fig. 7 with Fig. 8, it is noted that the accuracy of the estimation decreases as σ is increased. A comparison of Figs. 7 and 9 shows that the accuracy of the inverse analysis is improved when the sensors are located closer to the unknown wall heat flux.

In the final case, the unknown wall heat flux is assumed to depend on both X and τ

$$Q(X,\tau) = 14\sin\left(\frac{\pi}{100}X\right)\sin(10\pi\tau) \tag{22}$$

which is shown in Fig. 10. Figs. 11 and 12 are used to demonstrate the effects of measurement errors on the accuracy of the inverse analysis. The effects of the location of the sensors on the inverse solution are shown in Figs. 11 and 13. Overall, it can be seen that the estimation of the wall heat flux is good.

4. Conclusion

The inverse problem of determining the wall heat flux for unsteady turbulent forced convection between flat plates has been considered. The unknown wall heat flux to be estimated is a function of space and time. The temperature measurements within the flow are assumed available. The conjugate gradient method is applied to minimize the sum of square residuals between calculated and measured temperature. No prior information on the functional form of the wall heat flux is needed in the inverse method. The sensitivity problem can be solved only once for the iteration procedure by the conjugate gradient method. The performance of the present method is tested by numerical experiments. Eleven to twenty-five iterations are required to get the inverse solutions for the cases considered in this paper. The inverse solutions are accurate for both exact and noisy data.

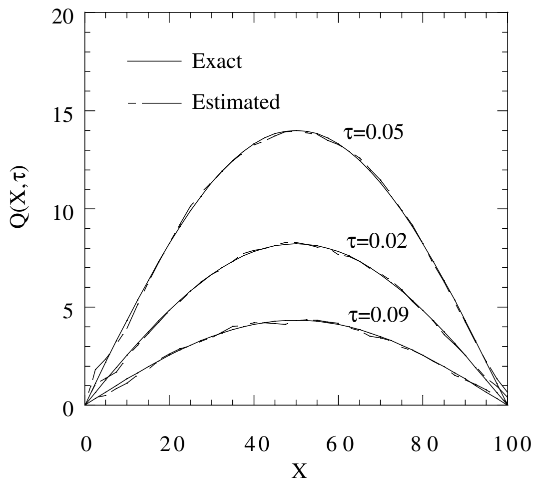

Fig. 13. Exact and estimated wall heat fluxes for $Y_1 = 0.95$ and $\sigma = 0.02$.

Acknowledgements

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